Probabilistic Hyperproperties with Rewards

NASA Formal Methods 2022

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[#]Michigan State University (USA), *RWTH Aachen (Germany), **TU-Wien (Austria)

May 27, 2022

MICHIGAN STATE UNIVERSITY

RWTH AACHEN UNIVERSITY

TU WIEN TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Trace Property vs Hyperproperty[1]

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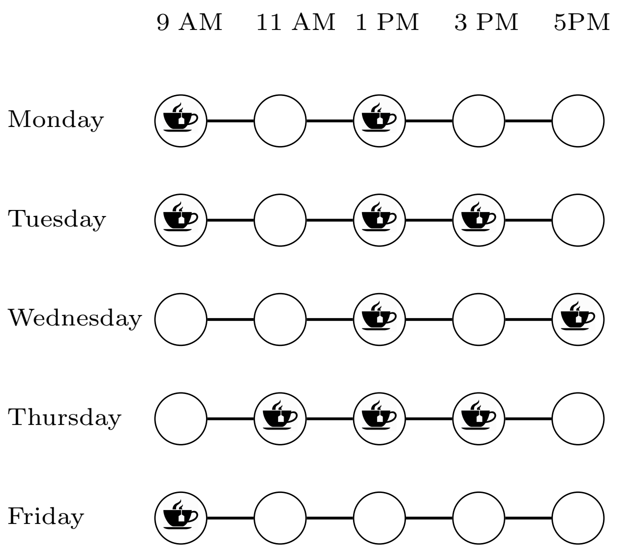

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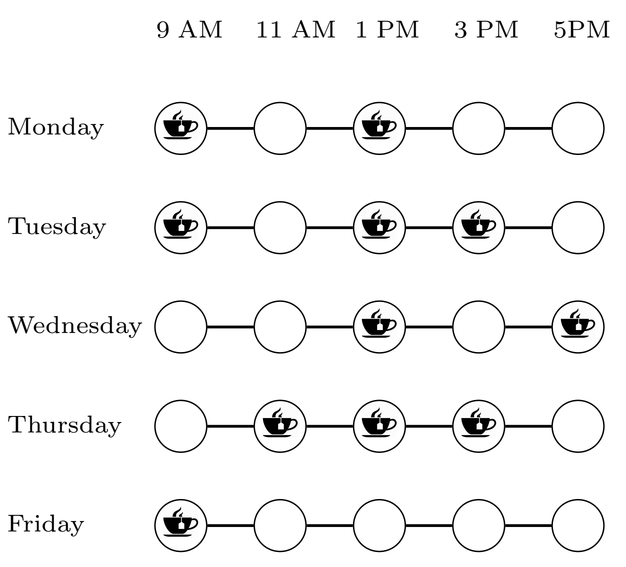

Trace property:

- I drink tea everyday - ◇ ☕ ✓

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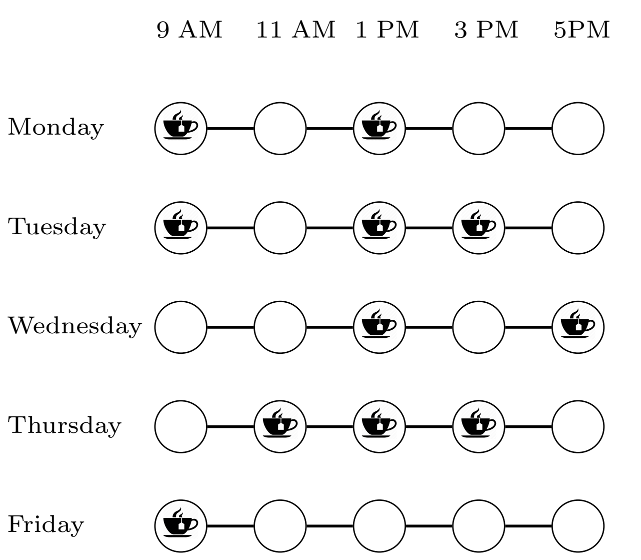

Trace property:

- I drink tea everyday - $\Diamond$ ☕ ✓
- I drink tea at the same time everyday - ✗

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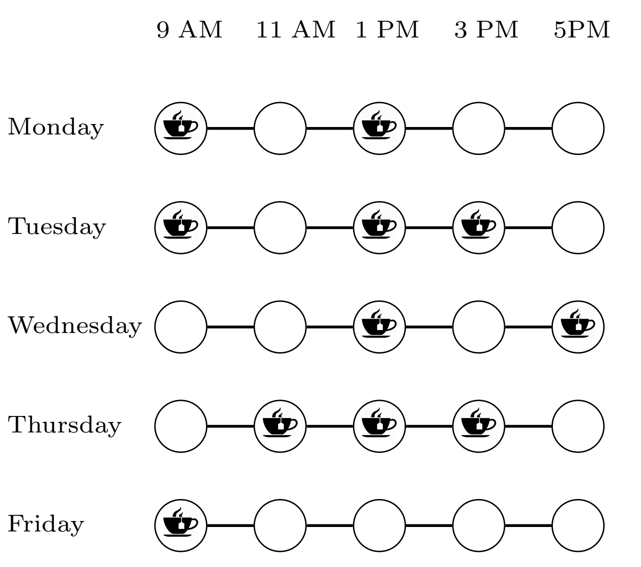

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Hyperproperty:

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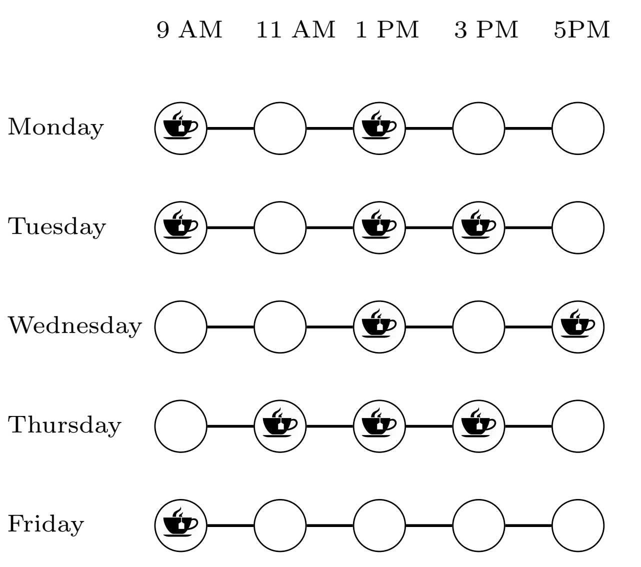

Trace property:

- I drink tea everyday - $\Diamond$ ☕ ✓
- I drink tea at the same time everyday - ✗

Hyperproperty:

I drink tea at the same time everyday

$\forall \pi . \forall \pi' . \Box$ (☕ $\leftrightarrow$ ☕) ✓

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Trace Property vs Hyperproperty (contd.)

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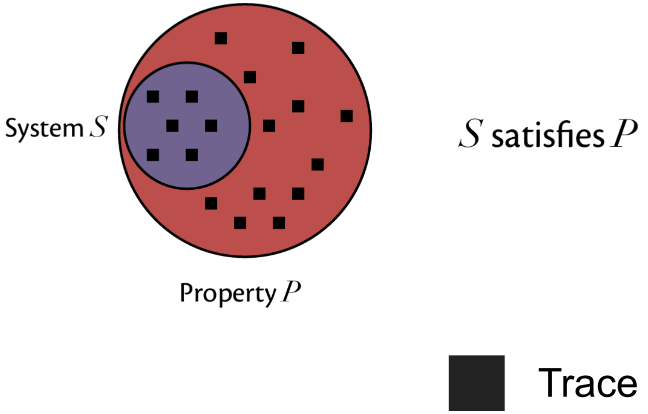
Fig. 1: Satisfaction of trace and hyper properties*



*Michael Clarkson's lecture notes on hyperproperties

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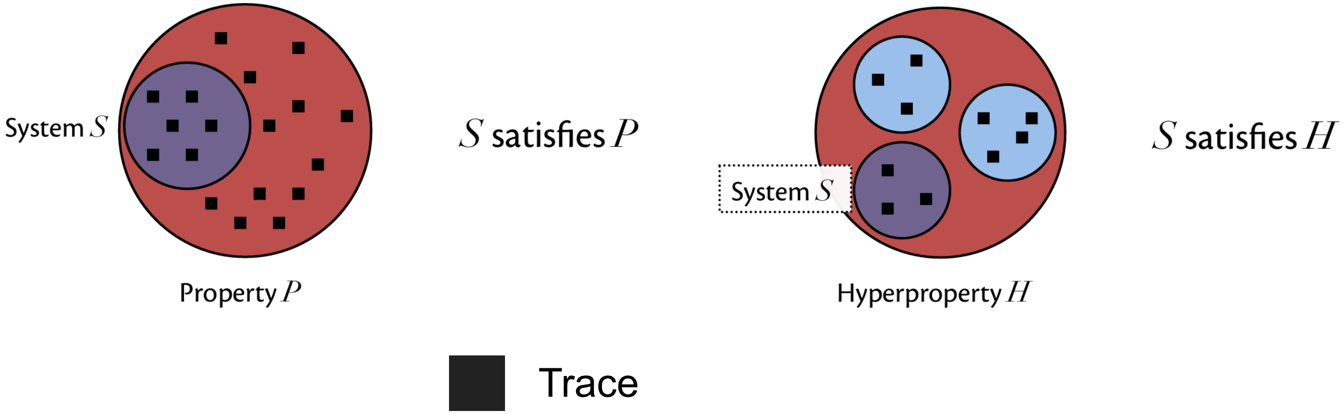

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Hyperproperties[1,2] in Action

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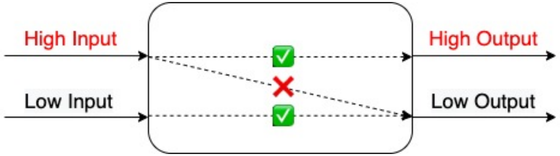
Hyperproperties[1,2] in Action

Non-interference[3]:



Should observe same low values for different high inputs

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Hyperproperties[1,2] in Action

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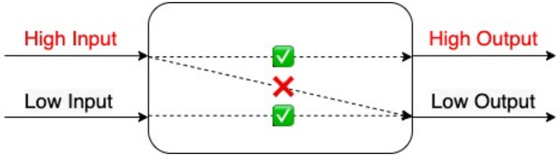

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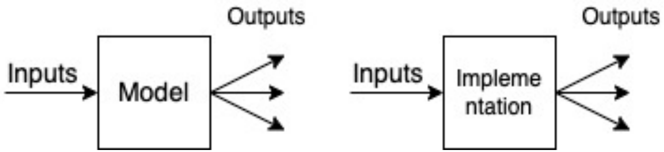
Conformance:



Should yield similar output after optimization / refactoring

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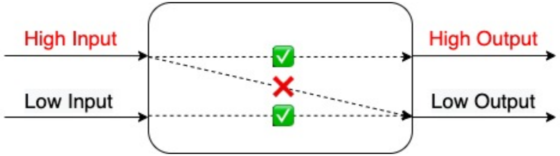

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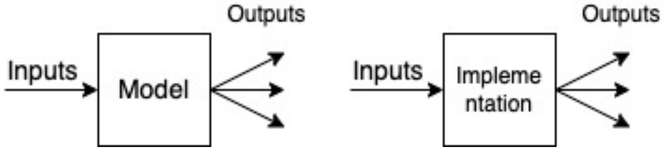

Should yield similar output after optimization / refactoring

Side-channel attacks:

```
void mexp(){
   // b is secret
  c=0;
  if (b(i) = 1){
    // changes to c
  }
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}
```

Should observe same execution times for different secrets

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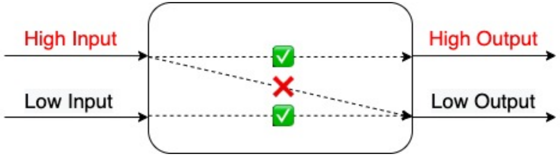

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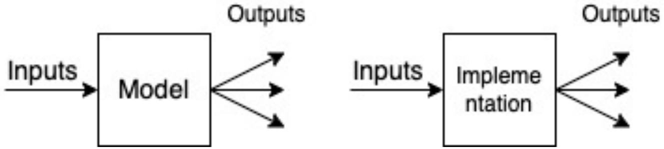

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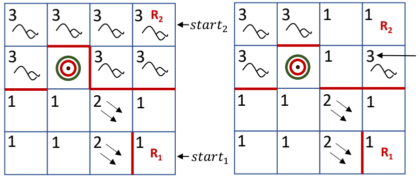
Robotics path planning[4]:



Finding paths: shortest, robustness, opaqueness

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Probabilistic Hyperproperties for DTMCs[1]

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- Motivation: Uncertainty and randomization.
- Probabilistic relation between traces.

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Fig 2: A Differential Privacy protocol

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$\boxed{\text{Flip a coin}}$

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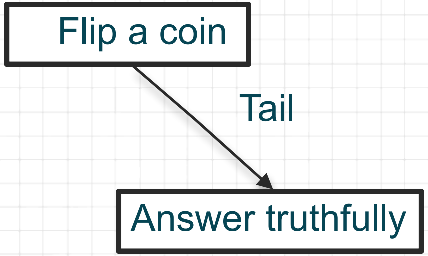

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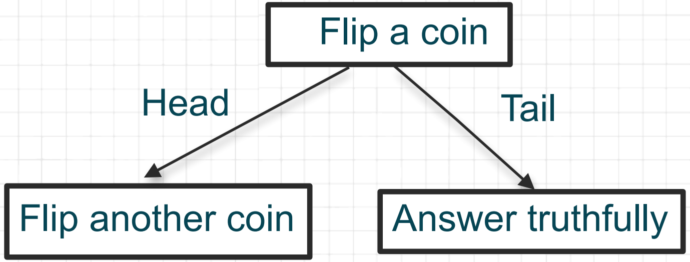

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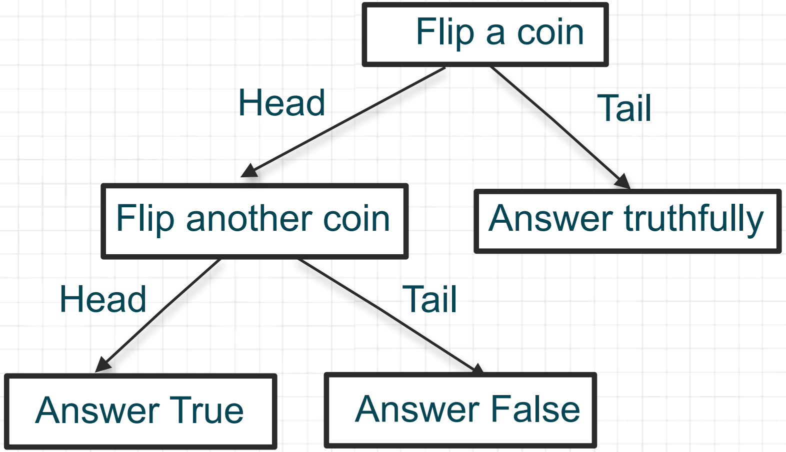

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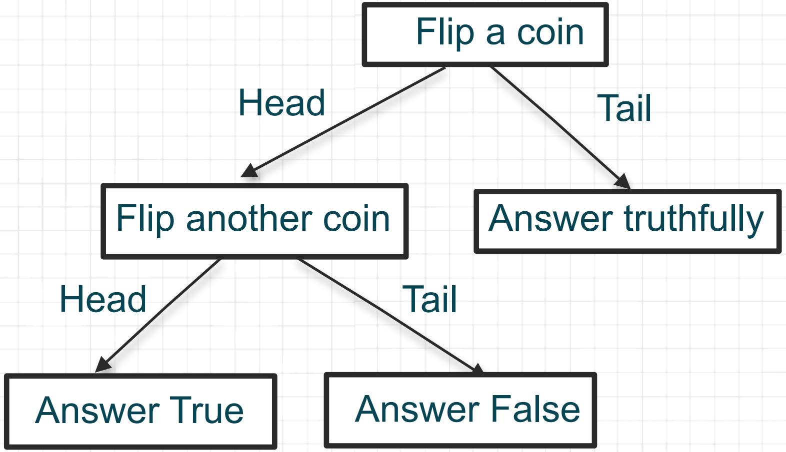

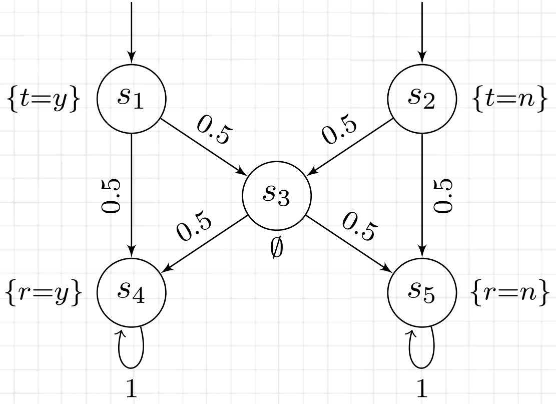

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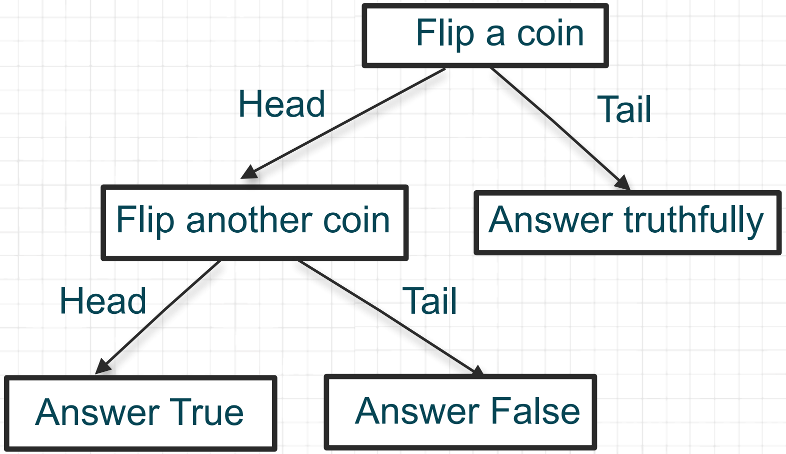

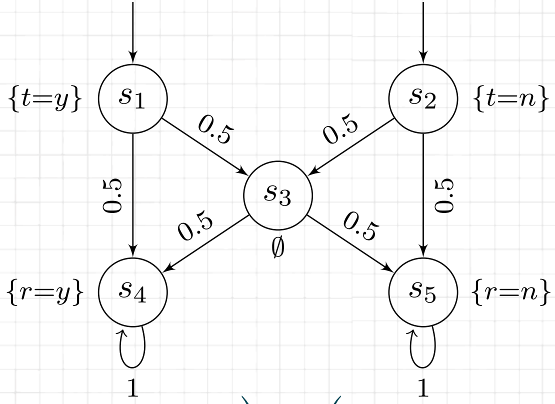

$$\psi = \forall s\,.\,\forall s'\,.\,\Big((\text{t=y})_s \wedge (\text{t=n})_{s'}\Big) \Rightarrow \Big(\mathbb{P}\,\Diamond(\text{r=y})_s \sim_\epsilon \mathbb{P}\Diamond(\text{r=y})_{s'}\Big)$$

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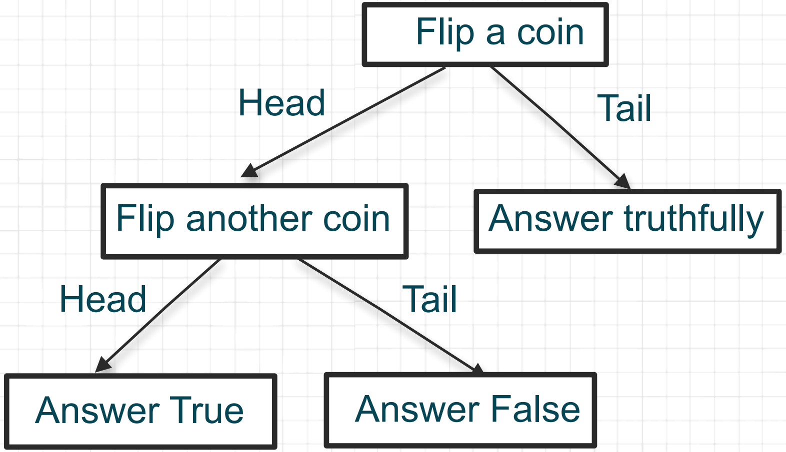

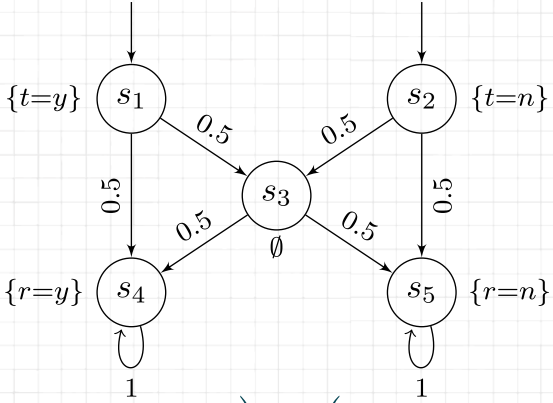

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Quantification over initial states

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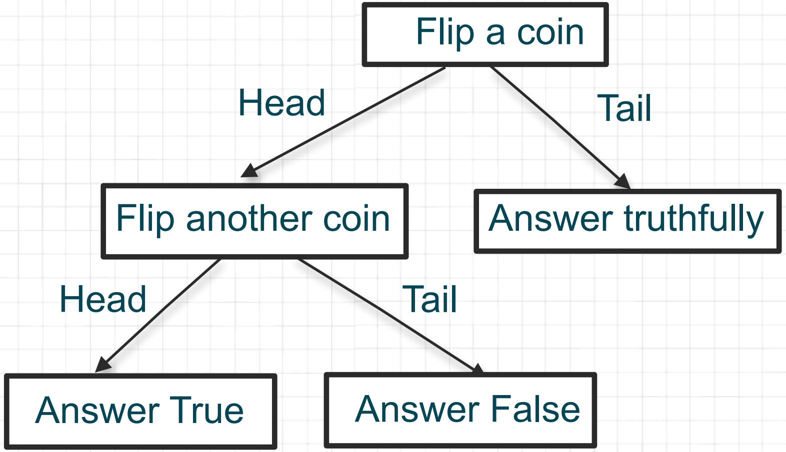

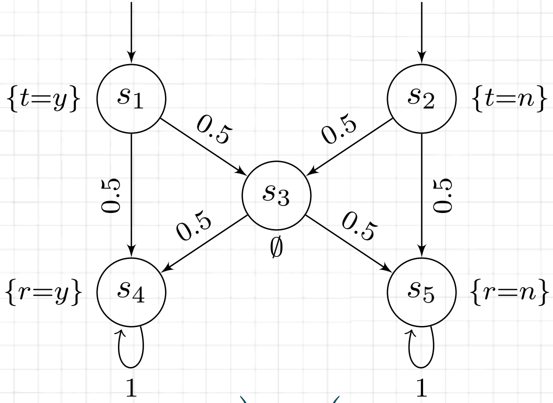

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Index propositions

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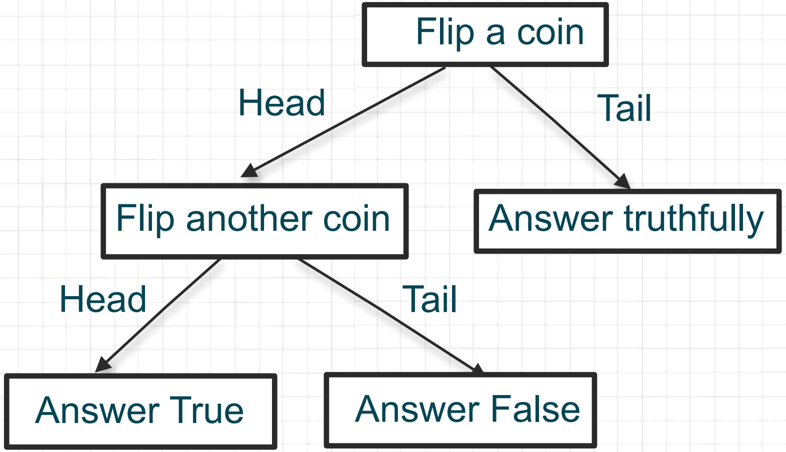

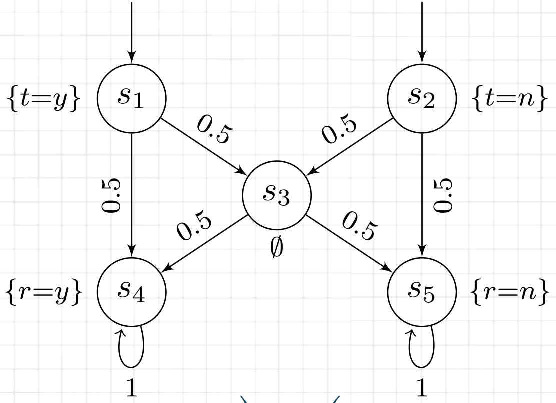

$$\psi = \underbrace{\forall s\,.\,\forall s'}_{\text{Quantification over initial states}}.\Big((t{=}y)_s \wedge (t{=}n)_{s'}\Big) \Rightarrow \Big(\underbrace{\mathbb{P}\,\Diamond(r{=}y)_s \sim_\epsilon \mathbb{P}\,\Diamond(r{=}y)_{s'}}_{\text{Temporal inside probability}}\Big)$$

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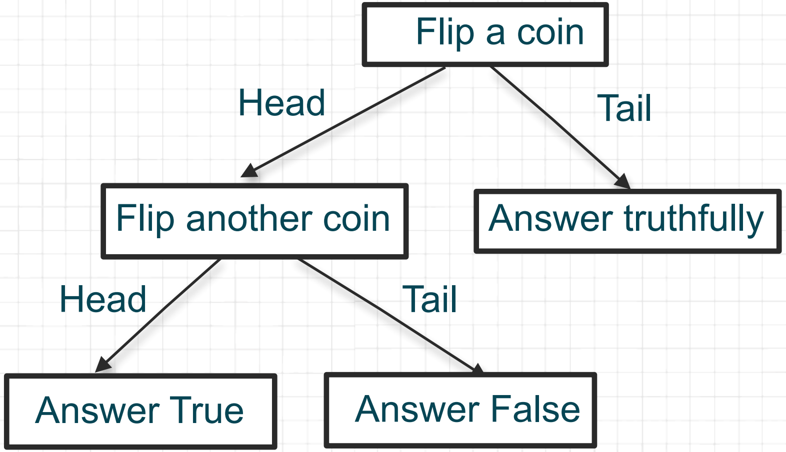

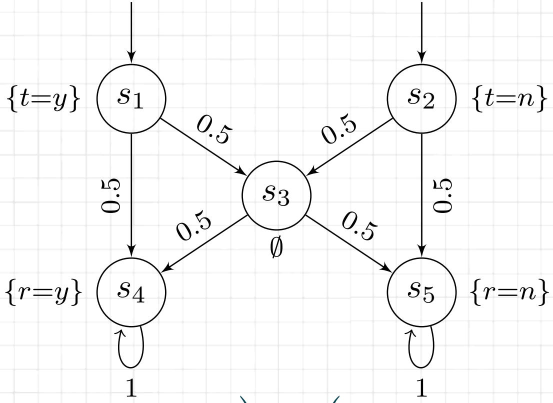

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Temporal inside probability

Cannot handle non-determinism!

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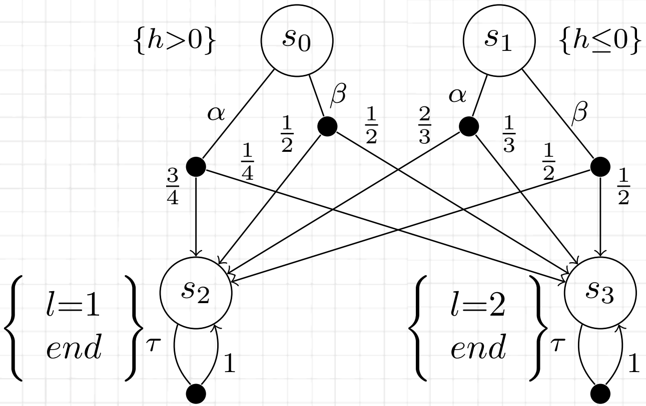

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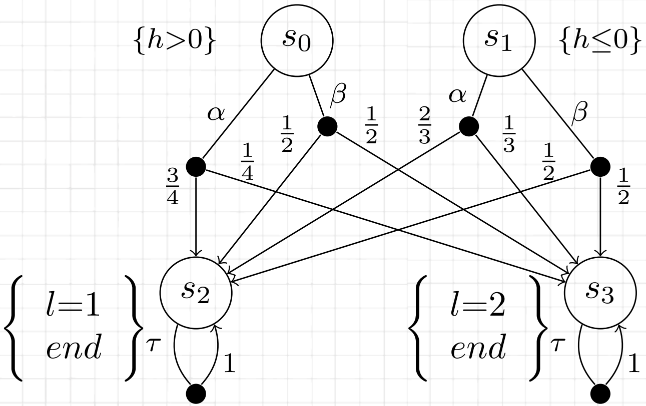

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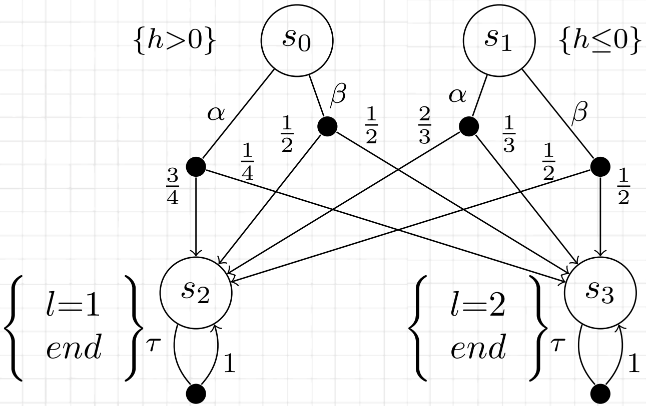

Scheduler quantifier

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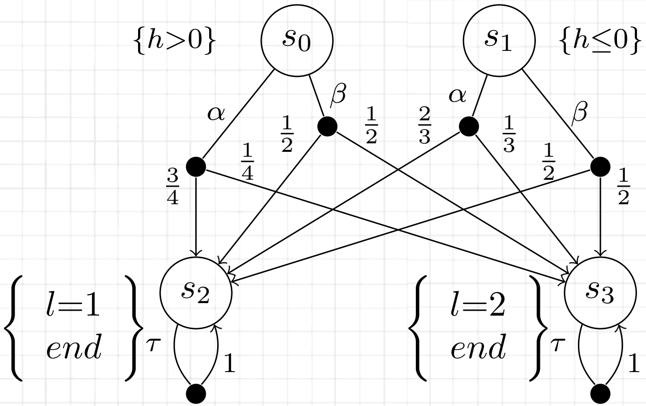

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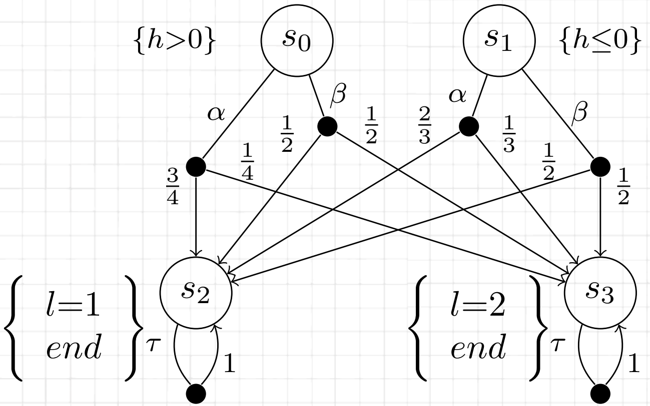
- Model checking problem for MDPs is undecidable.



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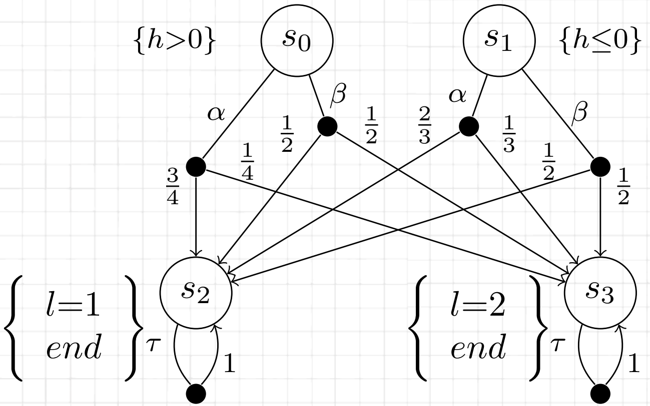


- Model checking problem for MDPs is undecidable.
- In [2], restricted schedulers to memoryless and non-probabilistic.

Scheduler quantifier

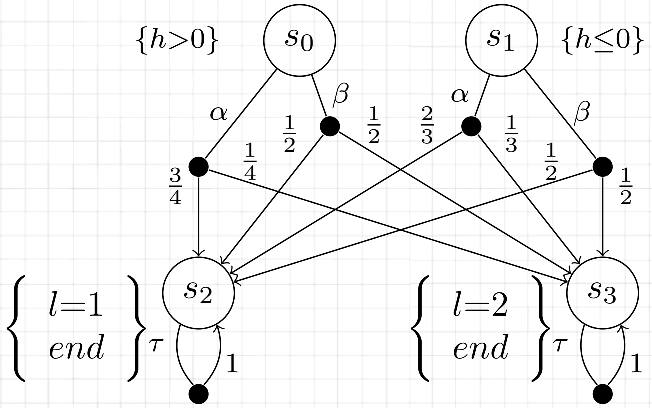
$$\exists sh\,.\,\forall s(sh)\,.\,\forall s'(sh)\,.\,\Big((h>0)_s \wedge (h\leq 0)_{s'}\Big) \Rightarrow \Big(\mathbb{P}\left(\Diamond(l=1)_s\right) = \mathbb{P}\left(\Diamond(l=1)_{s'}\right)\Big)$$

State quantifier scheduler

1. 'Probabilistic Hyperproperties of Markov Decision Processes', Dimitrova, Finkbeiner, Torfah, ATVA 2020.
2. 'Probabilistic Hyperproperties with Nondeterminism', Ábrahám, Bartocci, Bonakdarpour, Dobe, ATVA 2020.

Probabilistic Hyperproperties with Non-determinism[1,2]

- Argues over combination of schedulers.
- [1] introduces PHL extending HyperCTL* and has path quantification.
- [2] extends HyperPCTL and has state quantification.



- Model checking problem for MDPs is undecidable.
- In [2], restricted schedulers to memoryless and non-probabilistic.

Does not support reward models!

Scheduler quantifier

$$\exists sh\,.\,\forall s(sh)\,.\,\forall s'(sh)\,.\,\Big((h>0)_s \wedge (h\leq 0)_{s'}\Big) \Rightarrow \Big(\mathbb{P}\left(\Diamond(l=1)_s\right) = \mathbb{P}\left(\Diamond(l=1)_{s'}\right)\Big)$$

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Applications

Side-Channel timing leaks

Side-Channel timing leaks

Figure: Snippet of Modular exponentiation in RSA.

```
void mexp(){
  c = 0; d = 1; i = k;
  while (i >= 0){
    i = i-1; c = c*2;
    d = (d*d) % n;
    if (b(i) = 1){
      c = c+1;
      d = (d*a) % n;
    }
  }
}
```

Side-Channel timing leaks

Figure: Snippet of Modular exponentiation in RSA.

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Modeled as non-deterministic choice

Side-Channel timing leaks

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Modeled as non-deterministic choice

Lack of code in else causes failure

Side-Channel timing leaks

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  }
}
```

Modeled as non-deterministic choice

Lack of code in else causes failure

$$\forall \hat{\sigma} \,.\, \forall \hat{s}(\hat{\sigma}) \,.\, \forall \hat{s}'(\hat{\sigma}) \,.\, \left(\mathbb{R}_{\hat{s}}(\Diamond\ \mathsf{end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond\ \mathsf{end}_{\hat{s}'})\right)$$

Side-Channel timing leaks

Figure: Snippet of Modular exponentiation in RSA.

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Scheduler quantifier

Side-Channel timing leaks

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```
void mexp(){
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  while (i >= 0){
    i = i-1; c = c-1;
    d = (d*d) % n;
    if (b(i) = 1){
      c = c+1;
      d = (d*a) % n;
    }
  }
}
```

Modeled as non-deterministic choice

$$\forall \hat{\sigma} \,.\, \underbrace{\forall \hat{s}(\hat{\sigma}) \,.\, \forall \hat{s}'(\hat{\sigma})}_{\text{For all different combination for 'b'}} \,.\, \Big(\mathbb{R}_{\hat{s}}(\Diamond\, \mathsf{end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond\, \mathsf{end}_{\hat{s}'})\Big)$$

Scheduler quantifier

Lack of code in else causes failure

Side-Channel timing leaks

Figure: Snippet of Modular exponentiation in RSA.

```
void mexp(){
  c = 0; d = 1; i = k;
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```

Lack of code in else causes failure

Modeled as non-deterministic choice

$$\forall \hat{\sigma} . \forall \hat{s}(\hat{\sigma}) . \forall \hat{s}'(\hat{\sigma}) . \left(\mathbb{R}_{\hat{s}}(\Diamond\ \mathsf{end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond\ \mathsf{end}_{\hat{s}'})\right)$$

Scheduler quantifier

For all different combination for ‘b’

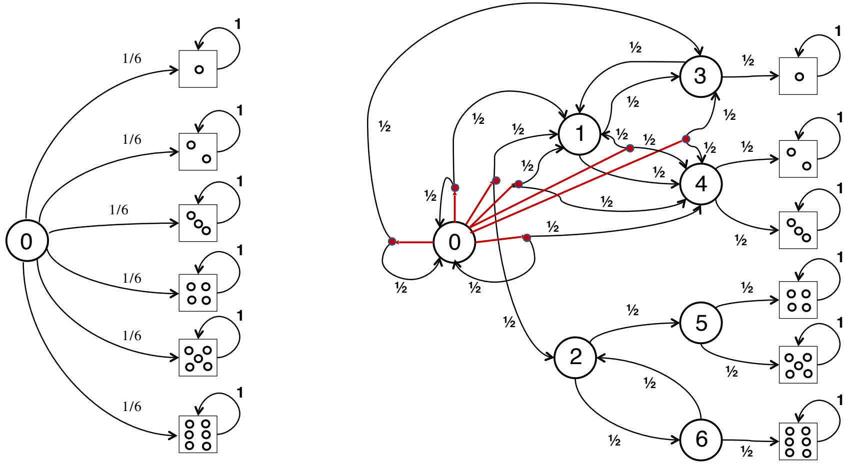
Ensure the code ends at the same time

Rewards helped us model time more efficiently

Probabilistic Conformance

Probabilistic Conformance

Figure: (Left) Model of a fair 6-sided dice. (Right) Model of the Knuth-Yao algorithm to simulate the dice.



Probabilistic Conformance

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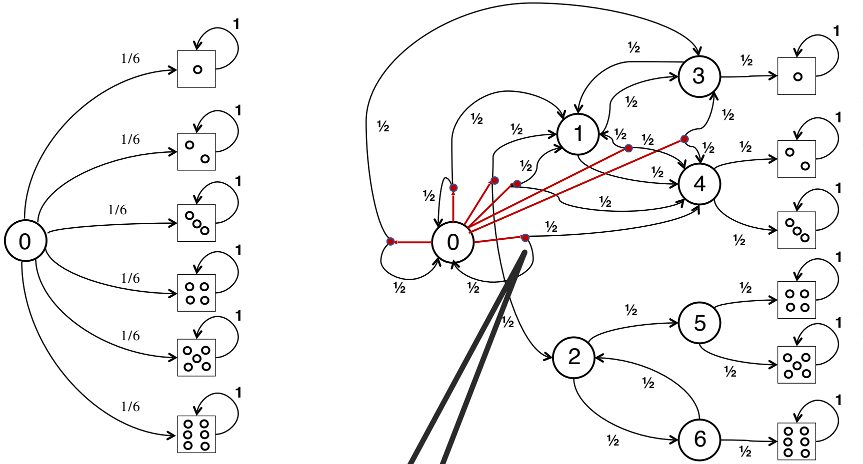
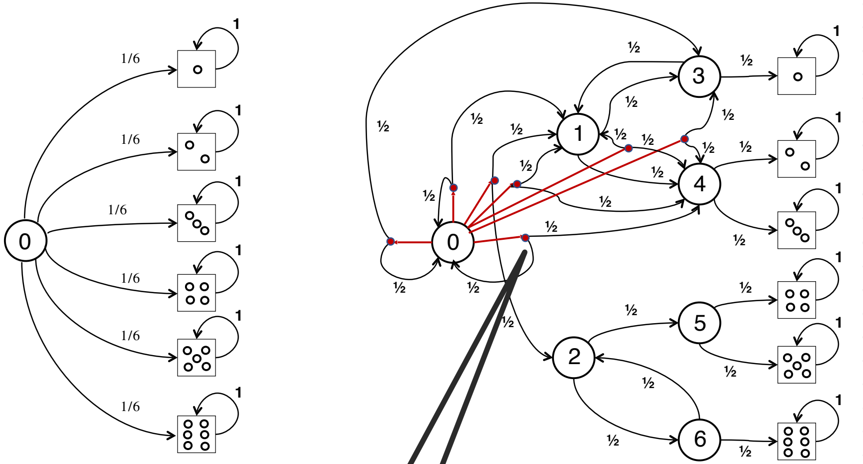


Modeled as non-deterministic choices from each state

Probabilistic Conformance

Figure: (Left) Model of a fair 6-sided dice. (Right) Model of the Knuth-Yao algorithm to simulate the dice.

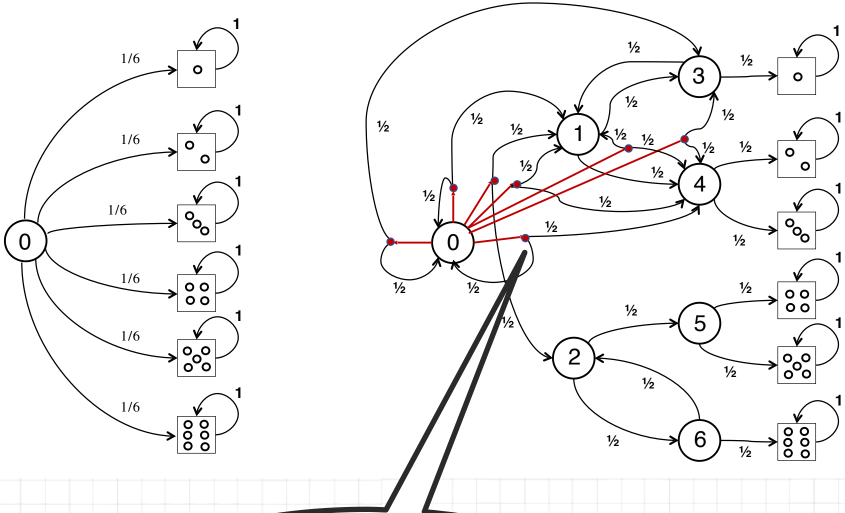


Modeled as non-deterministic choices from each state

$$\exists \hat{\sigma} . \forall \hat{s}(\hat{\sigma}) . \exists \hat{s}'(\hat{\sigma}) . \text{dieInit}_{\hat{s}} \rightarrow \left(\phi \wedge \mathbb{R}_{\hat{s}'}(F(\bigvee_{l=1}^{6} (die = l)_{\hat{s}'})) < 4 \right)$$

$$\phi = \text{coinInit}_{\hat{s}'} \wedge \bigwedge_{l=1}^{6} \left(\mathbb{P}(F(die = l)_{\hat{s}}) = \mathbb{P}(F(die = l)_{\hat{s}'}) \right)$$

Probabilistic Conformance



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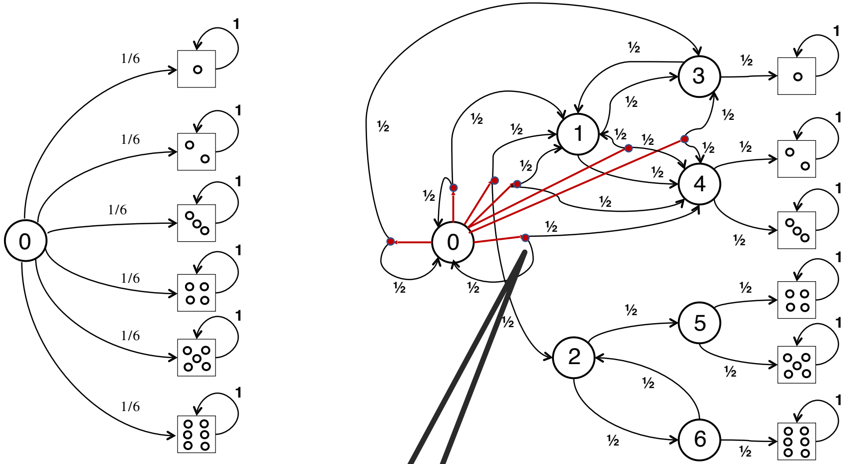
Probability distribution of die faces should be the same

Probabilistic Conformance

Figure: (Left) Model of a fair 6-sided dice. (Right) Model of the Knuth-Yao algorithm to simulate the dice.



Modeled as non-deterministic choices from each state

Limiting solutions to within 4 tosses!

$$\exists \hat{\sigma} \,.\, \forall \hat{s}(\hat{\sigma}) \,.\, \exists \hat{s}'(\hat{\sigma}) \,.\, \mathsf{dieInit}_{\hat{s}} \rightarrow \left(\phi \wedge \overbrace{\mathbb{R}_{\hat{s}'}(F(\bigvee_{l=1}^{6} (die = l)_{\hat{s}'})) < 4}^{} \right)$$

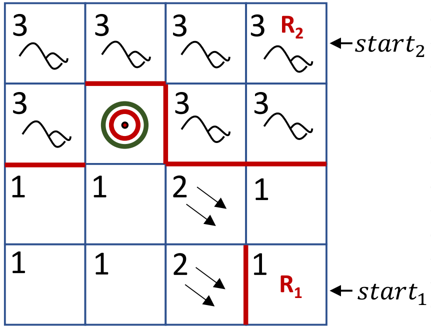
$$\phi = \mathsf{coinInit}_{\hat{s}'} \wedge \bigwedge_{l=1}^{6} \underbrace{\left(\mathbb{P}(F(die = l)_{\hat{s}}) = \mathbb{P}(F(die = l)_{\hat{s}'})\right)}_{}$$

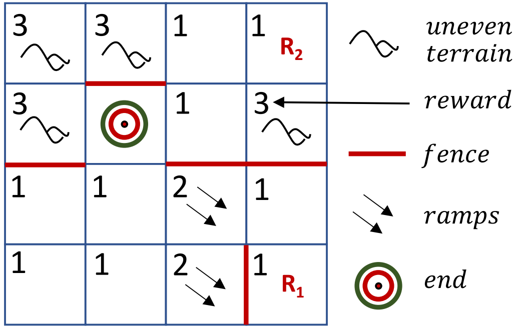
Probability distribution of die faces should be the same

Rewards helped us filter efficient solutions

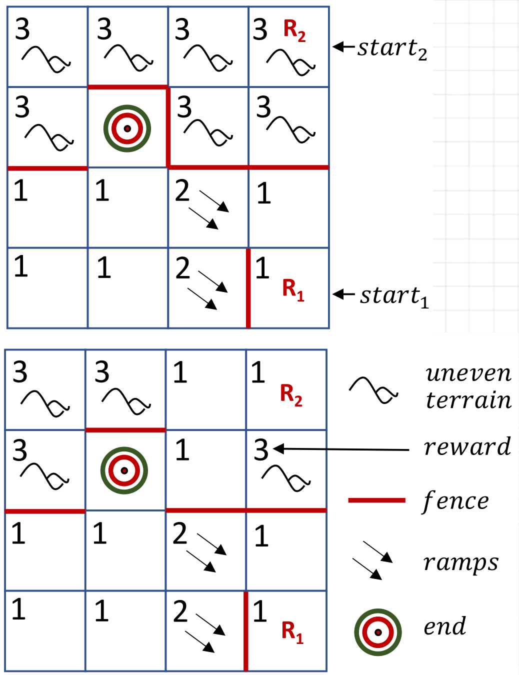
Multi-agent path planning

Multi-agent path planning





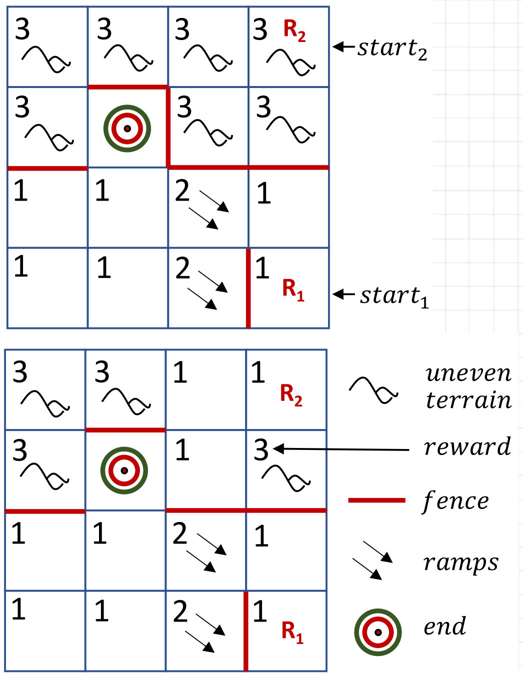
Multi-agent path planning



$$\varphi_{target} = \forall \hat{\sigma} . \forall \hat{s}(\hat{\sigma}) . \forall \hat{s}'(\hat{\sigma}) . \psi \rightarrow \left(\mathbb{R}_{\hat{s}}(\Diamond \mathsf{end}_{\hat{s}}) < \mathbb{R}_{\hat{s}'}(\Diamond \mathsf{end}_{\hat{s}'}) \right)$$

$$\psi = \left(\mathsf{start}_{1_{\hat{s}}} \wedge \mathsf{start}_{2_{\hat{s}'}} \wedge \mathbb{P}(\Diamond \mathsf{end}_{\hat{s}}) = 1 \wedge \mathbb{P}(\Diamond \mathsf{end}_{\hat{s}'}) = 1 \right)$$

Multi-agent path planning



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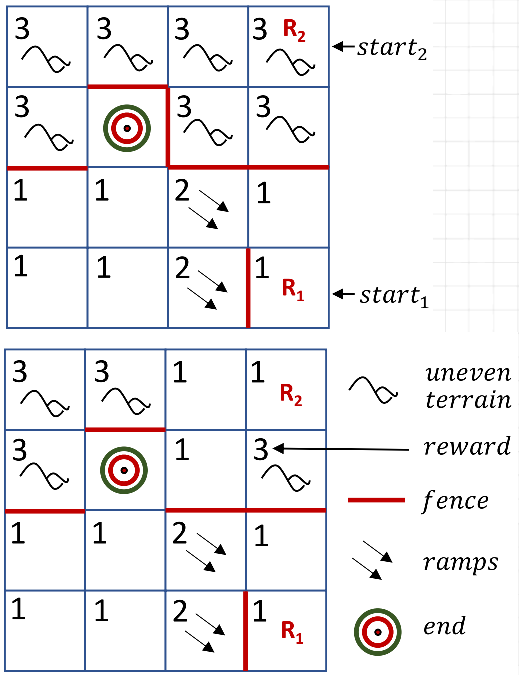
Ensures both robots reach goal state

Multi-agent path planning



Robot R1 should spend less energy

$$\varphi_{target} = \forall \hat{\sigma}\,.\,\forall \hat{s}(\hat{\sigma})\,.\,\forall \hat{s}'(\hat{\sigma})\,.\,\psi \rightarrow \left(\mathbb{R}_{\hat{s}}(\Diamond \mathsf{end}_{\hat{s}}) < \mathbb{R}_{\hat{s}'}(\Diamond \mathsf{end}_{\hat{s}'})\right)$$

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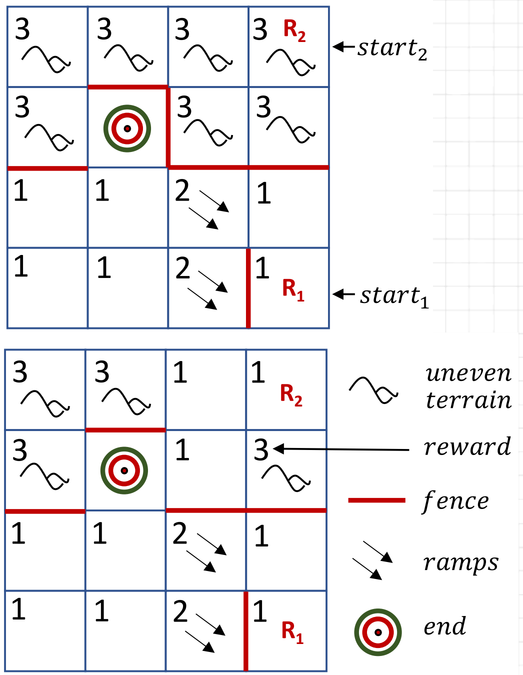
Multi-agent path planning

Figure: The maze on the top satisfies φ_{target}, while the bottom one violates it.



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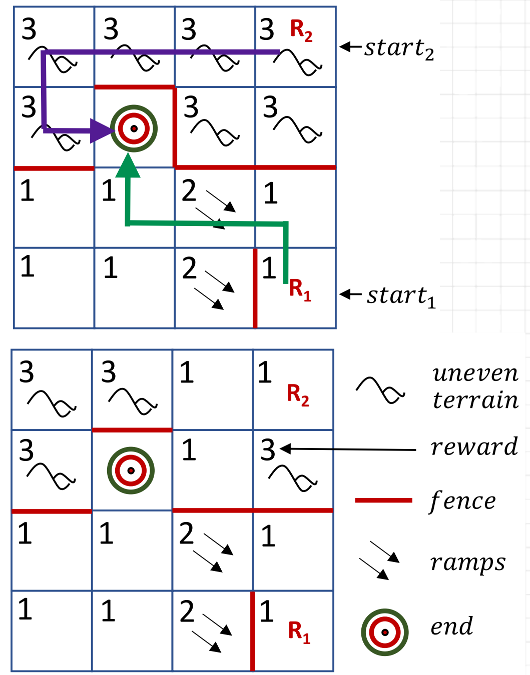

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Multi-agent path planning

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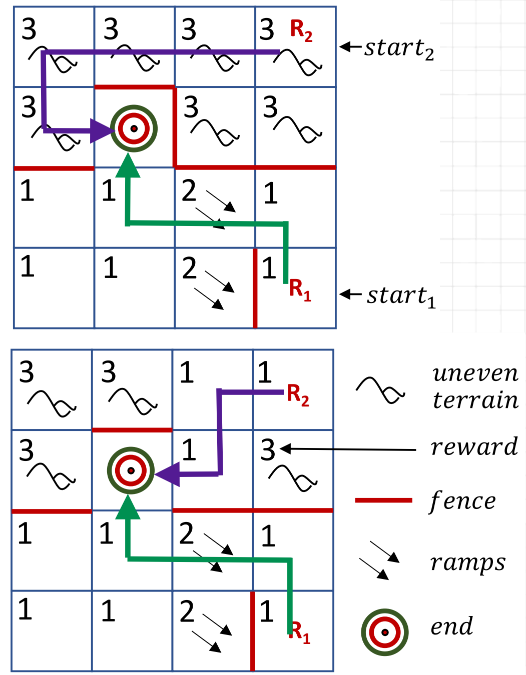

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Robot R1 should spend less energy

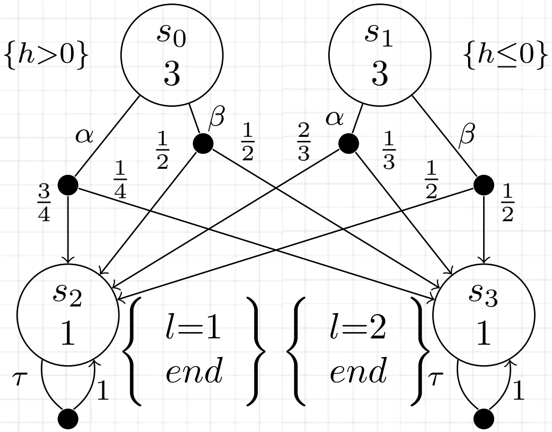
Ensures both robots reach goal state

Rewards helped us analyze cost of path planning

Methodology

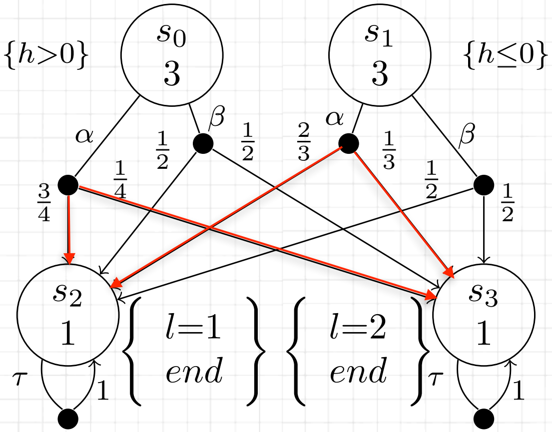
Probabilistic Hyperproperties with Rewards

Probabilistic Hyperproperties with Rewards



$$\exists \hat{\sigma}_1 \,.\, \exists \hat{\sigma}_2 \,.\, \forall \hat{s}(\hat{\sigma}_1) \,.\, \forall \hat{s}'(\hat{\sigma}_2) \,.\, \Big((h > 0)_{\hat{s}} \wedge (h \leq 0)_{\hat{s}'} \Big) \rightarrow$$
$$\Big(\mathbb{R}_{\hat{s}}(\Diamond \ \mathsf{end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond \ \mathsf{end}_{\hat{s}'}) \Big)$$

Probabilistic Hyperproperties with Rewards



$$\exists\hat{\sigma}_1 \,.\, \exists\hat{\sigma}_2 \,.\, \forall\hat{s}(\hat{\sigma}_1) \,.\, \forall\hat{s}'(\hat{\sigma}_2) \,.\, \Big((h > 0)_{\hat{s}} \land (h \leq 0)_{\hat{s}'}\Big) \rightarrow$$
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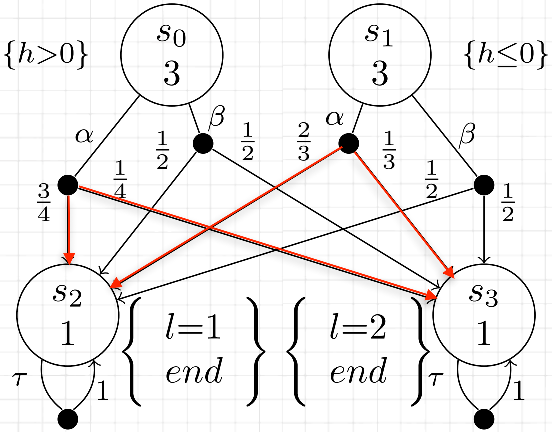
Probabilistic Hyperproperties with Rewards



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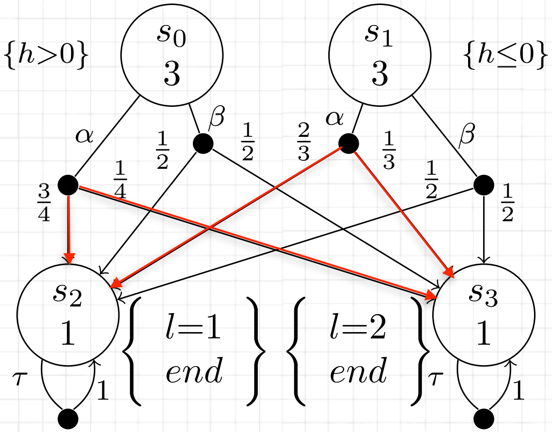
When $\hat{s} = s_0$

$$\mathbb{R}_{\hat{s}}(\Diamond\ \mathsf{end}_{\hat{s}}) = 3 + \frac{3}{4} * 1 + \frac{1}{4} * 1 = 4$$

When $\hat{s}' = s_1$

$$\mathbb{R}_{\hat{s}}(\Diamond\ \mathsf{end}_{\hat{s}}) = 3 + \frac{2}{3} * 1 + \frac{1}{3} * 1 = 4$$

Probabilistic Hyperproperties with Rewards



- Different Cases:

$$\exists \hat{\sigma}_1 . \exists \hat{\sigma}_2 . \forall \hat{s}(\hat{\sigma}_1) . \forall \hat{s}'(\hat{\sigma}_2) . \Big((h > 0)_{\hat{s}} \wedge (h \leq 0)_{\hat{s}'} \Big) \rightarrow \Big(\mathbb{R}_{\hat{s}}(\Diamond \text{ end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond \text{ end}_{\hat{s}'}) \Big)$$

When $\hat{s} = s_0$

$$\mathbb{R}_{\hat{s}}(\Diamond \text{ end}_{\hat{s}}) = 3 + \frac{3}{4} * 1 + \frac{1}{4} * 1 = 4$$

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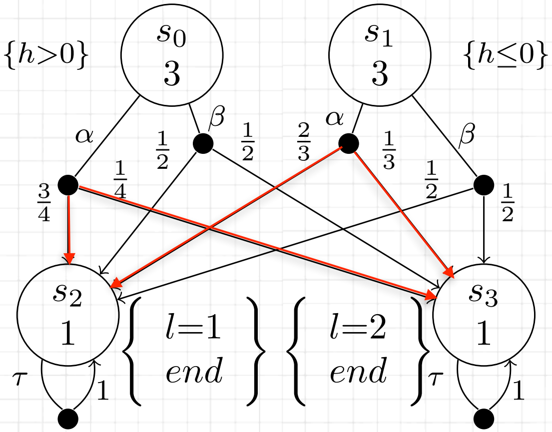
Probabilistic Hyperproperties with Rewards



- Different Cases:
  - ✓ Compare rewards across computation trees.

$$\mathbb{R}_{\hat{s}}(F\ \text{end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(F\ \text{end}_{\hat{s}'})$$

$$\exists\hat{\sigma}_1\,.\,\exists\hat{\sigma}_2\,.\,\forall\hat{s}(\hat{\sigma}_1)\,.\,\forall\hat{s}'(\hat{\sigma}_2)\,.\,\Big((h>0)_{\hat{s}} \wedge (h\leq 0)_{\hat{s}'}\Big) \rightarrow \Big(\mathbb{R}_{\hat{s}}(\Diamond\ \text{end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond\ \text{end}_{\hat{s}'})\Big)$$

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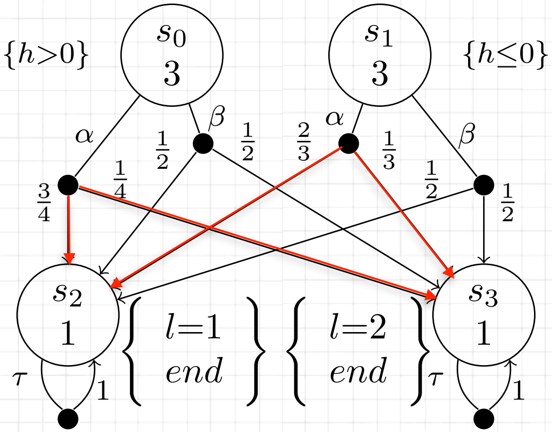
Probabilistic Hyperproperties with Rewards



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$$\Big(\mathbb{R}_{\hat{s}}(\Diamond \text{ end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(\Diamond \text{ end}_{\hat{s}'})\Big)$$

- Different Cases:
  - ✓ Compare rewards across computation trees.

    $$\mathbb{R}_{\hat{s}}(F \text{ end}_{\hat{s}}) = \mathbb{R}_{\hat{s}'}(F \text{ end}_{\hat{s}'})$$

  - ✓ Compute rewards in one tree until we reach a state in another.

    $$\mathbb{R}_{\hat{s}}\ (\text{good}_{\hat{s}}\ U \text{ end}_{\hat{s}'}) < \ 4$$

When $\hat{s} = s_0$

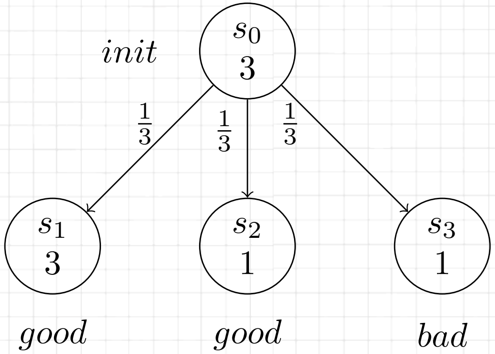
$$\mathbb{R}_{\hat{s}}(\Diamond \text{ end}_{\hat{s}}) = 3 + \frac{3}{4} * 1 + \frac{1}{4} * 1 = 4$$

When $\hat{s}' = s_1$

$$\mathbb{R}_{\hat{s}}(\Diamond \text{ end}_{\hat{s}}) = 3 + \frac{2}{3} * 1 + \frac{1}{3} * 1 = 4$$

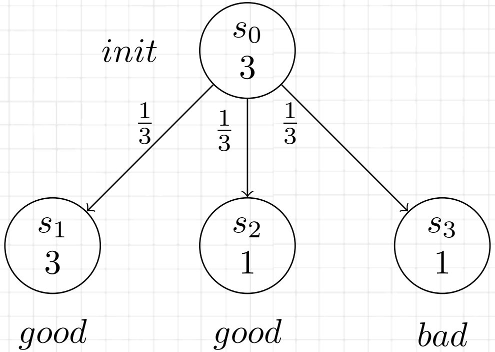
Undefinedness of Rewards

Undefinedness of Rewards



$\mathbb{R}(\bigcirc$ good$) = 3 + \frac{1}{3} * 3 + \frac{1}{3} * 1 + ? =$ undefined

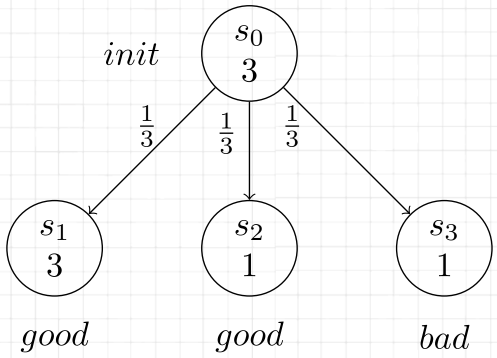
Undefinedness of Rewards



$$\mathbb{R}(\bigcirc\ \text{good}) = 3 + \frac{1}{3} * 3 + \frac{1}{3} * 1 + ? = \text{undefined}$$

Rewards is undefined if reachability probability is <1

Undefinedness of Rewards



$$\mathbb{R}(\bigcirc\ \text{good}) = 3 + \frac{1}{3} * 3 + \frac{1}{3} * 1 + ? = \text{undefined}$$

Rewards is undefined if reachability probability is <1

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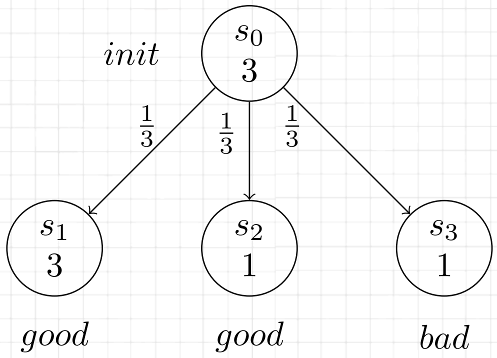

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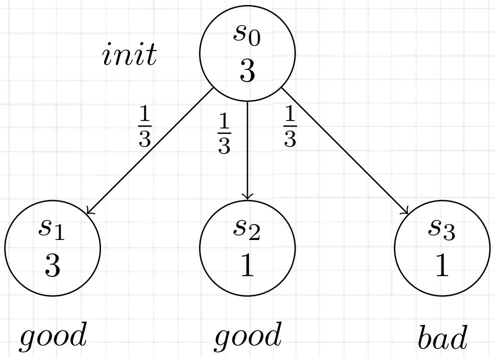

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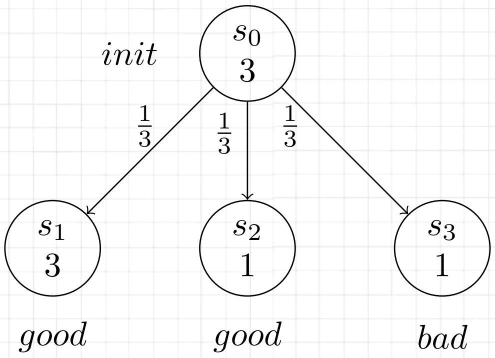

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Overview of Algorithm

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Input MDP satisfies the HyperPCTL formula

iff

SMT encoding is satisfied

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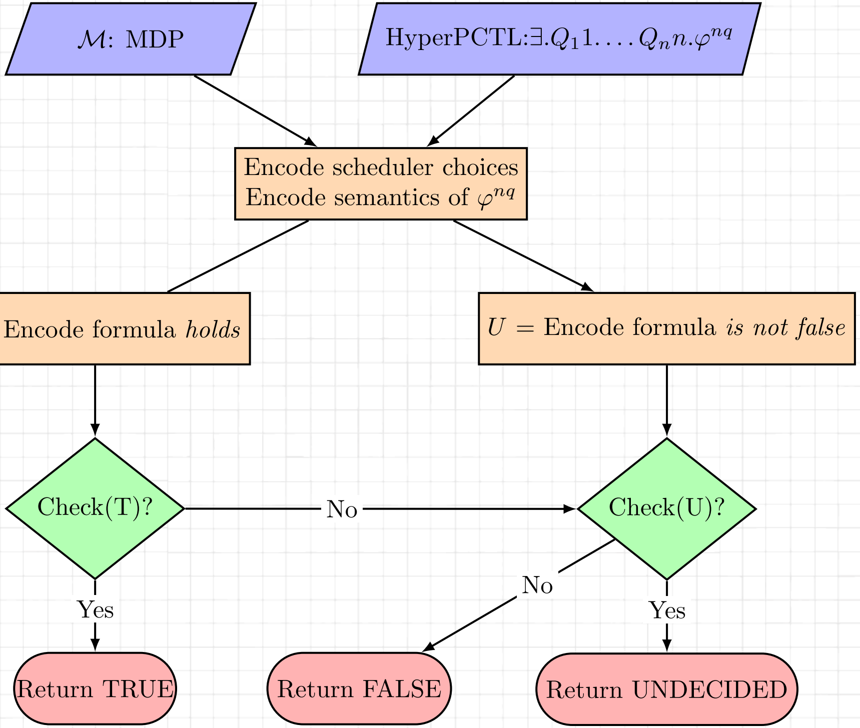
Algorithm for Existential Scheduler



Example of Encoding

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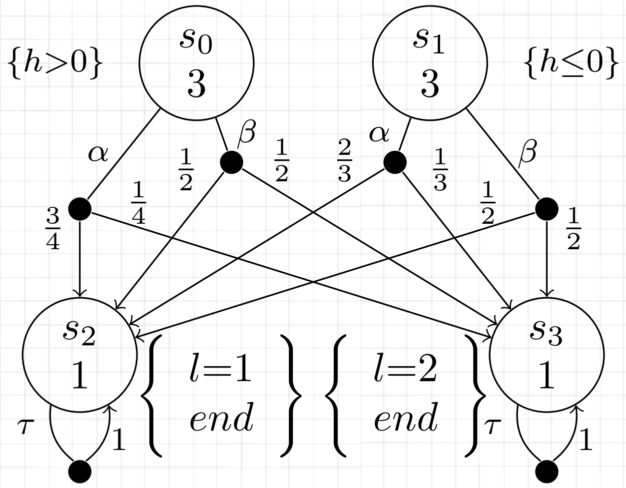

$$\varphi = \mathbb{R}_{\hat{s}}(\bigcirc\ \mathsf{end}_{\hat{s}})$$

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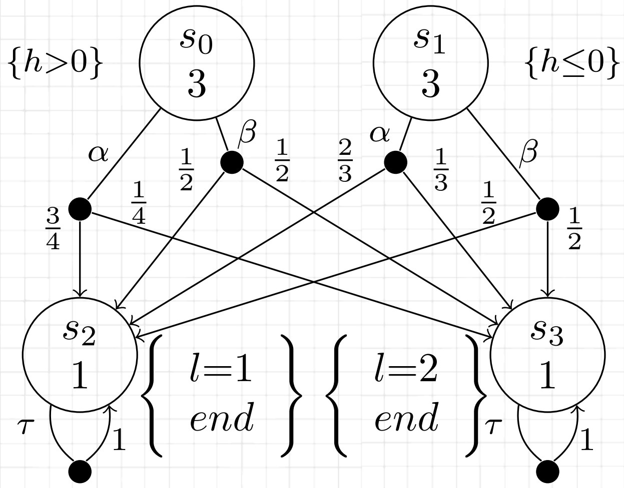

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Encoding for φ **=** ***Encoding for*** $\mathbb{P}(\bigcirc\ \textit{end}_{\hat{s}}) \bigwedge$

$\left(\textit{val}_{s_0,\mathbb{P}(\bigcirc\ \textit{end}_{\hat{s}})} \neq 1 \vee \neg \textit{def}_{s_0,\mathbb{P}(\bigcirc\ \textit{end}_{\hat{s}})}\right) \leftrightarrow \neg \textit{def}_{s_0,\varphi} \bigwedge \ldots$

$\left[\textit{def}_{s_0,\varphi} \wedge \textit{act}_{s_0} = \alpha\right] \rightarrow \left[\textit{val}_{s_0,\varphi} = 3 + (\frac{3}{4} * 1 + \frac{1}{4} * 1)\right] \bigwedge$

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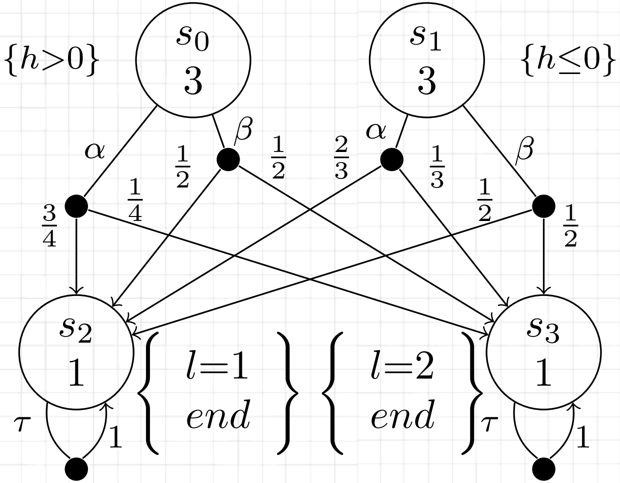

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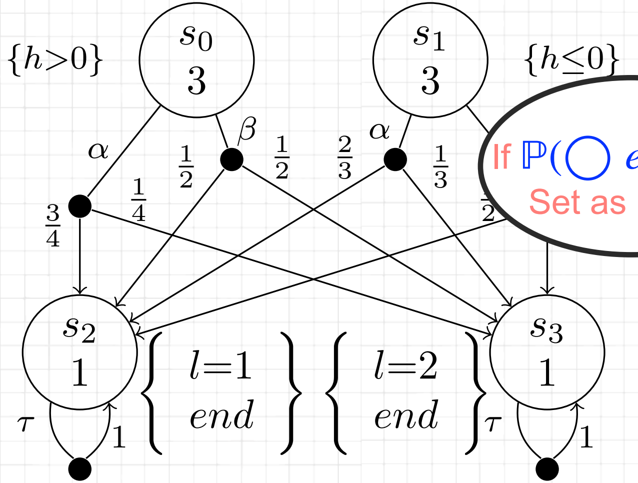

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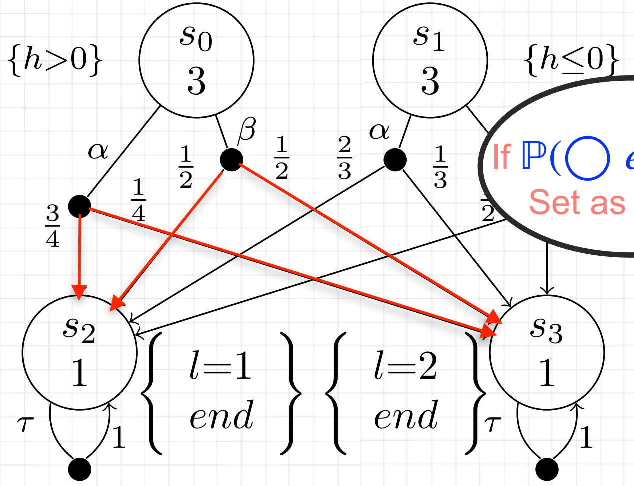

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Define case for each action and its transitions

Evaluation

Evaluation

Case study		VR	Running time (s) Encoding	Solving	Total	#SMT variables	#states	#transitions
TA	1-bit key	×	0.11	0.01	0.12	344	8	10
	16-bit key	×	16.41	3.69	20.10	19244	68	100
	30-bit key	×	143.49	44.64	188.13	62868	124	184
	45-bit key	×	774.53	1304.98	2079.51	137448	184	274
PC	s=(0)	✓	5.03	2.03	7.06	7281	20	186
	s=(0,1,2)	✓	6.66	8.91	15.57	7281	20	494
	s=(0,. . .,4)	✓	8.82	35	43.82	7281	20	802
	s=(0,. . .,6)	✓	11.64	53.05	64.69	7281	20	1110
RO	3x3	✓	0.87	0.05	0.92	2179	18	66
	3x3	×	0.93	0.05	0.98	2179	18	66
	4x4	✓	3.55	0.28	3.83	6561	32	160
	4x4	×	3.43	0.25	3.68	6561	32	148
	5x5	✓	13.07	0.5	13.57	15651	50	250
	5x5	×	13.19	0.98	14.17	15651	50	250
	6x6	✓	44.52	1.04	45.56	32041	72	398
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HS	$n = 3$	✓	0.1	0.01	0.11	489	8	28
	$n = 5$	✓	0.95	0.13	1.08	2369	32	244
IJ	$n = 3$	✓	0.08	0.01	0.09	169	7	21
	$n = 4$	✓	0.24	0.04	0.28	601	15	56
	$n = 5$	✓	0.89	0.33	1.22	2233	31	140
	$n = 6$	✓	3.93	19.39	23.32	8569	63	336

Experimental results: VR: Verification result. TA: Timing attack. PC: Probabilistic conformance. RO: Robotics example. HS: Herman's algorithm. IJ: Israeli-Jaflon's algorithm. ✓: the result is true. ×: the result is false.

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Implementation

Extended our tool HyperPROB[1] to accommodate restricted rewards.

1. 'HYPERPROB: A Model Checker for Probabilistic Hyperproperties', Dobe, Ábrahám, Bartocci, Bonakdarpour, FM 2021.